

This gif shows a construction of **truncated hyperbolic space**, or **neutered space**.

A hyperbolic finite volume and non-compact 3-manifold has universal cover the hyperbolic 3-space, and the fundamental group of that manifold acts on it but not geometrically, as the quotient is that non-compact manifold.

When we cut off the cusp, since the cusp retracts on a slice of itself, it does not change the fundamental group. Cutting the cusp, we see that the slice is a flat torus, whose universal cover is the flat plane.

This flat plane, in hyperbolic space, sits as a sphere centered at infinity, on the horizon: a horosphere. Cutting the cusp amounts to, at the universal cover level, removing an infinite collection of disjoint horoballs, which are the interior of those horospheres. Now the fundamental group acts on the truncated hyperbolic space geometrically.

The terminology “neutered space” was coined by Farb in 1994 and it is close to perfect: short, illustrative and easy to remember, it encodes in 2 words all the construction and most of the assumptions. The violence associated to that word, however, made it difficult to use and is at odds with the actual precision of the procedure: the horoballs removal is precisely encoded by the group action. In 1999, Bridson and Haefliger used the terminology “truncated hyperbolic space”, which is in fact quite accurate in the upper half plane model.

This gif has been made using notability on an iPad, hand drawing all 50 pictures individually.